SHORT COURSE:
INTRODUCTION TO COMPUTATIONAL GEOMECHANICS IN PETROLEUM ENGINEERING

By
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www.zamiran.net
Outline

- Introduction
- Numerical Analysis Methodology
- Wellbore Stability Analysis
- Flow from Boreholes in Biaxial Stress Fields
- Hydraulic Fracturing Simulation
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Graduated from Southern Illinois University Carbondale
Civil Engineering, Geotechnical
Dissertation: Earthquake Analysis of Retaining Walls
Geotechnical Engineer at Marino Engineering Associates, Inc.

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Forensic Engineering

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Pavement Testing
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MEA offers a number of petroleum engineering services, including:

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- Stability analysis of boreholes and optimization of mud type and pressure during drilling
- Geomechanical characterization of conventional and unconventional reservoirs
- Analysis and design of hydraulic fracturing methods
- Geophysical studies and image log analyses for provided reliable in-situ stress regime and mechanical rock characteristics specifically for mapping fluids contacts, estimating lithology, and characterizing reservoir fractures
- Geomechanical studies for in-situ thermal operations in oil reservoirs
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Numerical Analysis Methodology

FLAC, FLAC3D, UDEC, 3DEC, PFC
Numerical Modeling Procedure

- Observe
- Measure
- Explain
- Verify
- Results
Simple Definition of Modeling
Application of Computational Geomechanics

- Interpretation
- Design
- Prediction

Figure 3. Predicted and measured deflections of a tie-back wall (Carter et al. 2000).
Numerical Modeling Steps

Selection of representative cross-section
  Idealize the field conditions into a design X-section
  Plane strain vs. axisymmetrical models
Choice of numerical method and program
Defining the geometry
Assign constitutive model e.g. elastic, Mohr-Coulomb, etc.
Assign material properties
Generate grid/mesh for the domain
Assign boundary/loading conditions
Solve for initial condition
Problem alterations
Run the model
Obtain results
Interpret of results
Idealize Field Conditions to Numerical Modeling

- 3D modeling
- 2D modeling
  - Plain strain
    - No strain in the z direction
    - Structure or feature is relatively long
  - Axi-symmetry
Modeling Tools

- FLAC
- FLAC3D
- UDEC
- 3DEC
- PFC

- Command Keyword coding
You are not logged in.

I have an account  I want to create an account
## Demonstration Software

<table>
<thead>
<tr>
<th>DEMO NAME</th>
<th>LIMITS</th>
<th>64-BIT VERSION</th>
<th>32-BIT VERSION</th>
</tr>
</thead>
</table>
| **3DEC™ VERSION 5.2 Distinct-Element Modeling of Jointed and Blocky Material in 3D** | Version Number: 5.20  
Subversion: 260  
Maximum of 40 blocks and 1,000 zones. | Download Demo  
[🔗](#)                                      | Download Demo  
[🔗](#)                                      |
| **FLAC® VERSION 8.0 Explicit Continuum Modeling of Non-linear Material Behavior in 2D** | Version Number: 8.00  
Subversion: 449  
Maximum of 600 zones. | Download Demo  
[🔗](#)                                      | Download Demo  
[🔗](#)                                      |
| **FLAC/Slope™ VERSION 8.0 Explicit Continuum Modeling of Non-linear Material Behavior in 2D** | Version Number: 8.00  
Subversion: 444  
Maximum of 1000 zones. | Download Demo  
[🔗](#)                                      | Download Demo  
[🔗](#)                                      |
| **FLAC3D™ VERSION 6.0 Explicit Continuum Modeling of Non-linear Material Behavior in 3D** | Version Number: 6.00  
Subversion: 55  
Maximum of 600 zones and 100 structural elements.  
Project and Model save functionality and grid export are disabled. | Download Demo  
[🔗](#)                                      | Download Demo  
[🔗](#)                                      |
# Demonstration Software

<table>
<thead>
<tr>
<th>Demonstration Software</th>
<th>Version Number: 5.0</th>
<th>Download Demo</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PFC™ VERSION 5.0</strong></td>
<td>Subversion: 35</td>
<td></td>
</tr>
<tr>
<td>General Purpose Distinct-Element Modeling Framework</td>
<td>The PFC demo includes both PFC2D and PFC3D programs. Maximum of 1000 balls or 1000 clumps not exceeding 1000 balls, and 10 discrete fractures.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demonstration Software</th>
<th>Version Number: 8.00</th>
<th>Download Demo</th>
<th>Download Demo</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UDEC™ VERSION 6.0</strong></td>
<td>Subversion: 329</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distinct-Element Modeling of Jointed and Blocky Material in 2D</td>
<td>Maximum of 200 rigid blocks or 150 deformable blocks or 560 kB of memory.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FLAC/FLAC3D

- Large-strain simulation of continuia
- Groundwater flow, with full coupling to mechanical calculation
- Structural elements
- Thermal and creep calculations
- Dynamic analysis
- Two-phase fluid flow model
- User-defined constitutive models written in C++
- Built-in language (FISH) to add user-defined features (e.g., new constitutive models, new variables or new commands)
Serial Number: 212-001-1098-00001
Licensee: Itasca Consulting Group, Inc.
        Minneapolis, Minnesota, USA
Options: Dynamic
        Creep
        Thermal
        OppUdm
        TPFloe
Memory: 24.00 MBytes
Precision: Double

Project: <No name>
Title: <No title>
Sketch (0)
Gravity
FLAC3D
Stability analysis of jointed rock medium
Deep underground excavations
Blasting effects
Ground support reinforcement
Underground construction
Fluid-pressurized tunnels
Dams and dam foundations
Fluid flow though jointed rock (hydraulic fracturing)
Earthquake engineering
UDEC
UDEC - Dams and dam foundations
UDEC/3DEC- Tunnels
PFC2D/3D

- Particle Flow Code
- Thermal-mechanical coupling
- Add new physics using C++
- Available fluid dynamics add-on
FLAC
FLAC3D
Numerical Methods

- Continuum modelling
- Discontinuum modelling
- Limit equilibrium
- Hybrid/coupled modelling
Continuum Modelling

Finite element
- Plaxis
- Plaxis3D
- SIGMA/W
- Midas GTS
- RS

Finite difference
- FLAC
- FLAC3D

Discontinuum modelling

Distinct (discrete) element method:
Joints are treated as boundary conditions. Deformable blocks are discretized into internal constant-strain elements
- UDEC
- 3DEC

Particle flow codes
- PFC2D
- PFC3D

Hybrid/coupled modelling
- FLAC
- FLAC3D
- UDEC
- 3DEC

Limit equilibrium
- Slope/W
- Slide
 Finite Element Method

- Introduced from mechanical and structural analysis of beam, columns, frames, etc.
- Developed into continuous media $\Rightarrow$ soil
- Division of domain geometry $\Rightarrow$ finite element mesh
- Matrix operations for formulation
- Stiffness matrix generated
- Adjustment of field variables is made $\Rightarrow$ error term is minimized (energy)
Finite Difference Method

- Oldest & simplest technique
- No matrix operations
- Field variables
  - Stress or pressure
  - Displacement
  - Velocity
- Solution is done by time stepping (small interval of time)
- Each time step: grid values are updated
- Good method for:
  - Dynamic analysis
  - Large deformation analysis
Finite Difference Calculation Cycle

Equilibrium Equation
(Equation of Motion)

new velocities and displacements

Stress / Strain Relation
( Constitutive Equation)

new stresses or forces
Element vs. Grid

Element (FE)

Grid, Zone (FD)
Constitutive Relationships

- Elastic
  - Linear elasticity equations
  - Hooke's law

- Viscoelastic
  - Behave elastically
  - Also has damping (when the stress is applied and removed)

- Elasto-plastic
  - Applied stress is less than a yield value: elastic
  - More: plastic
Normal and Shear Stresses

\[ \mathbf{\sigma} = \begin{cases} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{cases} \]

- Normal stress in the x direction
- Normal stress in the y direction
- Normal stress in the z direction
- Shear stress on the xy plane
- Shear stress on the yz plane
- Shear stress on the zx plane

Recall that:

- \( \tau_{xy} = \tau_{yx} \)
- \( \tau_{yz} = \tau_{zy} \)
- \( \tau_{zx} = \tau_{xz} \)

Normal and shear stresses
Normal and Shear Strain

\[ \varepsilon_x = \frac{\partial u}{\partial x} \]  
Axial strain in the x-direction

\[ \varepsilon_y = \frac{\partial u}{\partial y} \]  
Axial strain in the y-direction

\[ \varepsilon_z = \frac{\partial w}{\partial z} \]  
Axial strain in the z-direction

\[ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \]  
Shear strain in the x-y plane

\[ \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \]  
Shear strain in the y-z plane

\[ \gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \]  
Shear strain in the z-x plane
Strain - Displacement

\[ \varepsilon = \left\{ \begin{array}{c} \varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{zx} \end{array} \right\} \]

- Axial strain in the x direction
- Axial strain in the y direction
- Axial strain in the z direction
- Shear strain in the xy plane
- Shear strain in the yz plane
- Shear strain in the zx plane

\[ \mathbf{u} = \left\{ \begin{array}{c} u \\
v \\
w \end{array} \right\} \]

- Displacement in the x direction
- Displacement in the y direction
- Displacement in the z direction

6 independent and unknown strains

3 independent and unknown displacements
Hooke's Law

**Stains from Hooke's Law**

\[
\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E}
\]

\[
\varepsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E} - \nu \frac{\sigma_z}{E}
\]

\[
\varepsilon_z = \frac{\sigma_z}{E} - \nu \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E}
\]

and

\[
\gamma_{xy} = \frac{\tau_{xy}}{G}
\]

\[
\gamma_{yz} = \frac{\tau_{yz}}{G}
\]

\[
\gamma_{zx} = \frac{\tau_{zx}}{G}
\]

**E =** elastic modulus

**\( \nu = \)** poisson's ratio

**G =** shear modulus

\[
G = \frac{E}{2(1+\nu)}
\]

**Stresses from Hooke's Law**

\[
\sigma_x = \frac{E}{(1+\nu)(1-2\nu)} \left[ (1-\nu)\varepsilon_x + \nu(\varepsilon_y + \varepsilon_z) \right]
\]

\[
\sigma_y = \frac{E}{(1+\nu)(1-2\nu)} \left[ (1-\nu)\varepsilon_y + \nu(\varepsilon_x + \varepsilon_z) \right]
\]

\[
\sigma_z = \frac{E}{(1+\nu)(1-2\nu)} \left[ (1-\nu)\varepsilon_z + \nu(\varepsilon_x + \varepsilon_y) \right]
\]

\[
E \equiv \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{\sigma}{\varepsilon}
\]

\[
G \overset{\text{def}}{=} \frac{\tau_{xy}}{\gamma_{xy}}
\]
Bulk Modulus

\[ K = -V \frac{\partial P}{\partial V} \]

P: pressure
V: volume
\( \partial P/\partial V \): partial derivative of pressure with respect to volume
## Elastic Correlations

<table>
<thead>
<tr>
<th></th>
<th>((G, \nu))</th>
<th>((E, \nu))</th>
<th>((K, \nu))</th>
<th>((K, E))</th>
<th>((M, G))</th>
</tr>
</thead>
<tbody>
<tr>
<td>47.1.1</td>
<td>(\frac{2G(1+\nu)}{3(1-2\nu)})</td>
<td>(\frac{E}{3(1-2\nu)})</td>
<td>(3K(1-2\nu))</td>
<td>(\frac{G(3M-4G)}{M-G})</td>
<td>(M - \frac{4G}{3})</td>
</tr>
<tr>
<td>47.1.2</td>
<td>(2G(1+\nu))</td>
<td>(3K(1-2\nu))</td>
<td>(\frac{3K(3K-E)}{9K-E})</td>
<td>(M - 2G)</td>
<td>(\lambda = \frac{2G\nu}{1-2\nu})</td>
</tr>
<tr>
<td>47.1.3</td>
<td>(G = \frac{E}{2(1+\nu)})</td>
<td>(\frac{3K(1-2\nu)}{2(1+\nu)})</td>
<td>(\frac{3KE}{9K-E})</td>
<td>(\nu = \frac{3K-E}{6K})</td>
<td>(\frac{M-2G}{2M-2G})</td>
</tr>
<tr>
<td>47.1.4</td>
<td>(M = \frac{2G(1-\nu)}{1-2\nu})</td>
<td>(\frac{E(1-\nu)}{(1+\nu)(1-2\nu)})</td>
<td>(\frac{3K(1-\nu)}{1+\nu})</td>
<td>(\frac{3K(3K+E)}{9K-E})</td>
<td>(\frac{M-2G}{2M-2G})</td>
</tr>
</tbody>
</table>
Elastic Model

The simplest representation of material behavior
Homogeneous
Isotropic
Continuous materials
Material that exhibit linear stress-strain behavior with no hysteresis on unloading

- Bulk modulus
- Shear modulus
- Modulus of elasticity
- Poisson's ratio
Mohr-Coulomb Model (Elastoplastic)

Conventional model used to represent shear failure in soils and rocks

Elastic portion
- Bulk modulus
- Shear modulus
- Modulus of elasticity
- Poisson's ratio

Plastic portion
- Cohesion
- Friction angle
- Dilation angle
- Tension
## Other Constitutive Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drucker-Prager</td>
<td>Soft clays with low friction angles</td>
</tr>
<tr>
<td>Ubiquitous-joint</td>
<td>Developed through Mohr-Coulomb solid, anisotropic</td>
</tr>
<tr>
<td>strain-hardening/softening</td>
<td>Nonlinear material softening and hardening behavior</td>
</tr>
<tr>
<td>Modified Cam-clay model</td>
<td>Soft clay</td>
</tr>
<tr>
<td>Hoek-Brown model</td>
<td>Intact rock and rock masses</td>
</tr>
</tbody>
</table>
Zone and Gridpoint

- Zone
- Gridpoint
- Velocities, Displacements
- Stresses
Boundary Condition

Boundary fixed in x direction

Typical boundary conditions
- Fixed in x direction
- Fixed in y direction
- Fixed in both directions
- Free in x and y directions (no boundary assigned)
Initial Conditions

- Effective vertical stress contours

**Initial Conditions that are generally considered:**
- Initial shear stresses
- Groundwater conditions
  - Hydrostatic water table
  - Flow gradient (non-steady state)
- For dynamic problems
  - Acceleration, velocity or stress time history

**Effective SYY-Stress Contours**
- Effective stress values:
  - -3.00E+05
  - -2.50E+05
  - -2.00E+05
  - -1.50E+05
  - -1.00E+05
  - -6.00E+04
  - 0.00E+00

**Initial groundwater conditions**
Boundary Conditions

- Fixed (X or Y) or both (B)
- Free

[X means fixed in x direction]
[B means fixed in both directions]
Applied Condition

- Velocity or displacement
- Stress or force

Yellow line with circle means force, velocity or stress has been applied to this surface.
Grids

Tunnel

Rock Slope with groundwater

Slope or Embankment

Concrete Diaphragm Wall
Assigning Material Models

Elastic Model
MODEL elastic and MODEL mohr-coul require that material properties be assigned via the PROPERTY command. For the elastic model, the required properties are (1) density; (2) bulk modulus; and (3) shear modulus.

\[
K = \frac{E}{3(1 - 2\nu)}
\]

\[
G = \frac{E}{2(1 + \nu)}
\]

\[
E = \frac{9KG}{3K + G}
\]

\[
\nu = \frac{3K - 2G}{2(3K + G)}
\]
Mohr-Coulomb plasticity model

(1) density;
(2) bulk modulus;
(3) shear modulus;
(4) friction angle;
(5) cohesion;
(6) dilation angle; and
(7) tensile strength.

grid 10,10
model elas j=6,10
prop den=2000 bulk=1e8 shear=.3e8 j=6,10
model mohr j=1,5
prop den=2500 bulk=1.5e8 shear=.6e8 j=1,5
prop fric=30 coh=5e6 ten=8.66e6 j=1,5
## Applying Boundary and Initial Conditions

<table>
<thead>
<tr>
<th>Command</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>APPLY</code></td>
<td><code>pressure</code></td>
</tr>
<tr>
<td></td>
<td><code>sxx</code></td>
</tr>
<tr>
<td></td>
<td><code>sxy</code></td>
</tr>
<tr>
<td></td>
<td><code>syy</code></td>
</tr>
<tr>
<td></td>
<td><code>xforce</code></td>
</tr>
<tr>
<td></td>
<td><code>yforce</code></td>
</tr>
<tr>
<td></td>
<td><code>xvelocity</code></td>
</tr>
<tr>
<td></td>
<td><code>yvelocity</code></td>
</tr>
<tr>
<td><code>FIX</code></td>
<td><code>pp</code></td>
</tr>
<tr>
<td></td>
<td><code>x</code></td>
</tr>
<tr>
<td></td>
<td><code>y</code></td>
</tr>
</tbody>
</table>
Sign Conventions
DIRECT STRESS – Positive stresses indicate tension; negative stresses indicate compression.

SHEAR STRESS
Figure 2.44  Distortion associated with positive and negative shear strain

Figure 2.45  Mechanical pressure: (a) positive; (b) negative
PORE PRESSURE – Fluid pore pressure is positive in compression. Negative pore pressure indicates fluid tension.

GRAVITY – Positive gravity will pull the mass of a body downward (in the negative y-direction).
Negative gravity will pull the mass of a body upward.

Table 2.5  Systems of units – mechanical parameters

<table>
<thead>
<tr>
<th></th>
<th>SI</th>
<th>Imperial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>m</td>
<td>ft</td>
</tr>
<tr>
<td>Density</td>
<td>kg/m³</td>
<td>slugs/ft³</td>
</tr>
<tr>
<td>Force</td>
<td>N</td>
<td>lbf</td>
</tr>
<tr>
<td>Stress</td>
<td>Pa</td>
<td>lbf/ft²</td>
</tr>
<tr>
<td>Gravity</td>
<td>m/sec²</td>
<td>ft/sec²</td>
</tr>
<tr>
<td>Stiffness*</td>
<td>Pa/m</td>
<td>lbf/ft³</td>
</tr>
</tbody>
</table>

* Stiffness refers to normal and shear stiffnesses at interfaces.

where
1 bar = 10^6 dynes/cm² = 10^5 N/m² = 10^5 Pa;
1 atm = 1.013 bars = 14.7 psi = 2116 lbf/ft² = 1.01325 × 10^5 Pa;
1 slug = 1 lbf - s²/ft = 14.59 kg;
1 snail = 1 lbf - s²/in; and
1 gravity = 9.81 m/s² = 981 cm/s² = 32.17 ft/s².
### Table 2.6  Systems of units — groundwater flow parameters

<table>
<thead>
<tr>
<th></th>
<th>SI</th>
<th>Imperial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water Bulk Modulus</td>
<td>Pa</td>
<td>bar</td>
</tr>
<tr>
<td>Water Density</td>
<td>kg/m³</td>
<td>10⁶ g/cm³</td>
</tr>
<tr>
<td>Permeability</td>
<td>m³ sec/kg</td>
<td>10⁻⁶ cm³ sec/g</td>
</tr>
<tr>
<td>Intrinsic Permeability</td>
<td>m²</td>
<td>cm²</td>
</tr>
<tr>
<td>Hydraulic Conductivity</td>
<td>m/sec</td>
<td>cm/sec</td>
</tr>
</tbody>
</table>

**NOTE:**  
FLAC permeability $\equiv$ intrinsic permeability (in cm²) $\times 9.9 \times 10^{-2}$  
FLAC permeability is the *mobility coefficient* (coefficient of pore pressure term in Darcy’s law).
Saving/Restoring Problem State

<table>
<thead>
<tr>
<th>Function</th>
<th>Command</th>
</tr>
</thead>
</table>
| Grid Generation                  | GRID
|                                  | GENERATE
|                                  | INITIAL               |
| Boundary/Initial Conditions      | APPLY
|                                  | FIX                    |
|                                  | INITIAL               |
| Material Model & Properties      | MODEL                  |
|                                  | PROPERTY               |
| Initial Equilibrium (with gravity)| STEP
|                                  | SOLVE                  |
|                                  | SET gravity            |
| Perform Alterations              | MODEL                  |
|                                  | PROPERTY               |
|                                  | APPLY                  |
|                                  | FIX                    |
|                                  | FREE                   |
| Save/Restore Problem State       | SAVE                   |
|                                  | RESTORE                |
Outline

- Numerical Analysis Methodology
- Wellbore Stability Analysis
- Flow from Boreholes in Biaxial Stress Fields
- Hydraulic Fracturing Simulation
3D Wellbore Stability Analysis

FLAC3D

- Part I: Isotropic medium
- Part II: Closed end wellbore
- Part III: Anisotropic medium
- Part IV: Coupled mechanical-fluid analysis
## Geometry Modeling

**Table 1.2** Summary of primitive mesh shapes (continued)

<table>
<thead>
<tr>
<th>Shape</th>
<th>Name</th>
<th>Keyword</th>
<th>Reference Points</th>
<th>Size Entries</th>
<th>Dimension Entries</th>
<th>Fill</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial Cylinder</td>
<td>radcylinder</td>
<td>12</td>
<td>4</td>
<td>4</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Cylindrical Shell</td>
<td>cshell</td>
<td>10</td>
<td>4</td>
<td>4</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Cylinder Intersection</td>
<td>cylint</td>
<td>14</td>
<td>5</td>
<td>7</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Tunnel Intersection</td>
<td>tunint</td>
<td>17</td>
<td>5</td>
<td>7</td>
<td>Yes</td>
<td></td>
</tr>
</tbody>
</table>
## Geometry Modeling

### Table 1.2 Summary of primitive mesh shapes

<table>
<thead>
<tr>
<th>Shape</th>
<th>Name</th>
<th>Keyword</th>
<th>Reference Points</th>
<th>Size Entries</th>
<th>Dimension Entries</th>
<th>Fill</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brick</td>
<td>brick</td>
<td>8</td>
<td>3</td>
<td>0</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>Degenerate Brick</td>
<td>dbrick</td>
<td>7</td>
<td>3</td>
<td>0</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>Wedge</td>
<td>wedge</td>
<td>6</td>
<td>3</td>
<td>0</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>Uniform Wedge</td>
<td>uwedge</td>
<td>6</td>
<td>3</td>
<td>0</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>Pyramid</td>
<td>pyramid</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>No</td>
<td></td>
</tr>
</tbody>
</table>
### Table 1.2  Summary of primitive mesh shapes

<table>
<thead>
<tr>
<th>Shape</th>
<th>Name</th>
<th>Keyword</th>
<th>Reference Points</th>
<th>Size Entries</th>
<th>Dimension Entries</th>
<th>Fill</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pyramid</td>
<td>pyramid</td>
<td></td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>No</td>
</tr>
<tr>
<td>Tetrahedron</td>
<td>tetrahedron</td>
<td></td>
<td>4</td>
<td>3</td>
<td>0</td>
<td>No</td>
</tr>
<tr>
<td>Cylinder</td>
<td>cylinder</td>
<td></td>
<td>6</td>
<td>3</td>
<td>0</td>
<td>No</td>
</tr>
<tr>
<td>Radial Brick</td>
<td>radbrick</td>
<td></td>
<td>15</td>
<td>4</td>
<td>3</td>
<td>Yes</td>
</tr>
<tr>
<td>Radial Tunnel</td>
<td>radtunnel</td>
<td></td>
<td>14</td>
<td>4</td>
<td>4</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Geometry Modeling

Figure 1.4  Brick mesh – brick
Geometry Modeling

Figure 1.10  Cylindrical mesh – cylinder
Figure 1.13  Radially graded mesh around cylindrical-shaped tunnel – radcylinder
Part I- Wellbore stability analysis in isotropic medium

new
gen zone radcyl group ub &
p0 0 0 0 p8 .08 0 0 p1 .4 0 0 p6 .4 .4 0 p3 0 .4 0 p9 0 .08 0 &
p2 0 0 -1.2 p10 .08 0 -1.2 p4 .4 0 -1.2 p7 .4 .4 -1.2 p5 0 .4 -1.2 p11 0 .08 -1.2 &
size 3 40 18 18 fill group ubhole ratio 1 1 1 1.15
Geometry Modeling

gen zone reflect dd 0 dip 90 origin 0 0 0 ; check origin 0 -0.5 0
gen zone reflect dd 90 dip 90

- dip: angle with xy plane
- dd: angle with xz plan
- origin x0, y0, z0
group mb range z -1.2 -.6 group ub
group mbhole range z -1.2 -.6 group ubhole

;---------------------------------------------------------------------PROPETY---------------------------------------------------------------------
model elastic range group ub
model elastic range group ubhole
model elastic range group mb
model elastic range group mbhole
;
prop s=1.84e9 b=3.19e9 range group ub
prop s=1.84e9 b=3.19e9 range group ubhole
prop s=8.02e9 b=12.62e9 range group mb
prop s=8.02e9 b=12.62e9 range group mbhole
Boundary Conditions

;---------------------------------------------FIXITIES---------------------------------------------
fix x range x -.39 -.41
fix x range x .39 .41
fix y range y .39 .41
fix y range y -.39 -.41
fix z range z -1.21 -1.19
fix z range z -.01 .01
Initial Stresses

;----------------------------------INITIAL STRESSES----------------------------------;
;-----ub
ini szz -76.0e6 range group ub
ini sxx -74.0e6 range group ub
ini syy -60.0e6 range group ub
ini szz -76.0e6 range group ubhole
ini sxx -74.0e6 range group ubhole
ini syy -60.0e6 range group ubhole
;-----mb
ini szz -76.0e6 range group mb
ini sxx -56.0e6 range group mb
ini syy -56.0e6 range group mb
ini szz -76.0e6 range group mbhole
ini sxx -56.0e6 range group mbhole
ini syy -56.0e6 range group mbhole
Initial Densities

;-------------------------------------INITIAL DENSITIES-------------------------------------
ini density=2300  range group ub
ini density=2300  range group ubhole
ini density=2600  range group mb
ini density=2600  range group mbhole
Analysis

set g 0 0 -9.81
solve
ini xdis 0 ydis 0 zdis 0;
model null range group ubhole
model null range group mbhole
solve
model mohr range group ub
model mohr range group mb;
prop s=1.84e9 b=3.19e9 f=35 c=12.e6 range group ub
prop s=8.02e9 b=12.62e9 f=37.3 c=15.63e6 range group mb
solve
save IUM.sav
Part II- Wellbore stability with closed end

Analyze the wellbore with the closed end
The transversely isotropic model takes a plane of isotropy into consideration. Let the axis of rotational symmetry, normal to the plane of isotropy, correspond to the local 3' axis. This axis is a principal direction of elasticity. Also, any two perpendicular directions 1', 2', which are principal directions of elasticity, can be selected in the isotropic plane. With this convention, the transversely isotropic model may be considered as a particular case of the orthotropic model for which

\[
\begin{align*}
E_1 &= E_2 \\
E' &= E_3 \\
G_{13} &= G_{23} \\
\nu_{13} &= \nu_{23} \\
G_{12} &= \frac{E_1}{2(1 + \nu_{12})}
\end{align*}
\]

- Young’s moduli in the plane of isotropy
- Young’s moduli in the direction normal to the plane of isotropy
- Poisson’s ratio characterizing lateral contraction in the plane of isotropy when tension is applied in this plane
- Poisson’s ratio characterizing lateral contraction in the plane of isotropy when tension is applied in the direction normal to it
- Shear modulus for the plane of isotropy
- Shear modulus for any plane normal to the plane of isotropy
The cross shear modulus, $G_{13}$, for anisotropic elasticity must be determined. Lekhnitskii (1981) suggests the following equation, based on laboratory testing of rock:

$$G_{xy} = \frac{E_x E_y}{E_x (1 + 2\nu_{xy}) + E_y}$$  \hspace{1cm} (1.28)
The transversely isotropic model takes a plane of isotropy into consideration.

**Transversely Isotropic Elastic – MODEL mechanical anisotropic**

- **dd**: dip direction [degrees] of plane of isotropy
- **density**: mass density, \( \rho \)
- **dip**: dip angle [degrees] of plane of isotropy
- **e1**: Young’s modulus in the plane of isotropy, \( E_1 \)
- **e3**: Young’s modulus normal to the plane of isotropy, \( E_3 \)
- **g13**: shear modulus of any plane normal to the plane of isotropy, \( G_{13} \)
- **nu12**: Poisson’s ratio characterizing lateral contraction in the plane of isotropy when tension is applied in the plane, \( \nu_{12} \)
- **nu13**: Poisson’s ratio characterizing lateral contraction in the plane of isotropy when tension is applied normal to the plane, \( \nu_{13} \)
Part III- Wellbore stability in anisotropic medium

;-----------------------------------PROPETY-----------------------------------
model anisotropic range group ub
model anisotropic range group ubhole
model anisotropic range group mb
model anisotropic range group mbhole
;
prop dip 0 dd 90 e1 1e10 e3 7.0e9 g13 2.92e9 nu12 0.20 nu13 0.17 range group ub
prop dip 0 dd 90 e1 1e10 e3 7.0e9 g13 2.92e9 nu12 0.20 nu13 0.17 range group ubhole
prop dip 0 dd 90 e1 19.92e9 e3 19.93e9 g13 7.94e9 nu12 0.24 nu13 0.24 range group mb
prop dip 0 dd 90 e1 19.92e9 e3 19.93e9 g13 7.94e9 nu12 0.24 nu13 0.24 range group mbhole
Part IV: Coupled mechanical/fluid analysis

config fluid

model fl_isotropic range group ub
model fl_isotropic range group ubhole
model fl_isotropic range group mb
model fl_isotropic range group mbhole

prop poros=0.35 perm=.4e-15 range group ub
prop poros=0.35 perm=.4e-15 range group ubhole
prop poros=0.25 perm=2e-15 range group mb
prop poros=0.25 perm=2e-15 range group mbhole

ini fmod=1e6 fdens=800

ini pp 60e6 range group ub
ini pp 60e6 range group ubhole
ini pp 42e6 range group mb
ini pp 42e6 range group mbhole

set fluid off mech on
## Table 2.7 Systems of units – groundwater flow parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SI</th>
<th>Imperial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water bulk modulus</td>
<td>Pa</td>
<td>bar</td>
</tr>
<tr>
<td>Water density</td>
<td>kg/m³</td>
<td>$10^6$ g/cm³</td>
</tr>
<tr>
<td>Permeability</td>
<td>m³/sec/kg</td>
<td>$10^{-6}$ cm sec/g</td>
</tr>
<tr>
<td>Intrinsic permeability</td>
<td>m²</td>
<td>cm²</td>
</tr>
<tr>
<td>Hydraulic conductivity</td>
<td>m/sec</td>
<td>cm/sec</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### NOTE:

- **FLAC³D** permeability (in SI units) ≡ intrinsic permeability (in cm²) × $9.9 \times 10^{-2}$
- **FLAC³D** permeability is the *mobility coefficient* (coefficient of pore pressure term in Darcy’s law).
Outline

- Numerical Analysis Methodology
- Wellbore Stability Analysis
- Flow from Boreholes in Biaxial Stress Fields
- Hydraulic Fracturing Simulation
Flow from Boreholes in Biaxial Stress Fields

UDEC
Flow from Boreholes in Biaxial Stress Fields

- A borehole is located in a rock mass
- Containing two orthogonal sets of joints
- Subjected to a biaxial in-situ stress state
- Fluid is injected into the borehole at a constant flow rate

**Scope:**
- Modeling flow into fractured rock
- Evaluate the influence of the in-situ stress state on flow into the joints
Problem Properties

- **In-situ stress field (σ: total stress)**
  \[ \sigma_{xx} = -25 \text{ MPa} \]
  \[ \sigma_{yy} = -20 \text{ MPa} \]
  \[ p = 10 \text{ MPa} \]

- **Intact rock block properties**
  - density 2500 kg/m\(^3\)
  - bulk modulus 66.667 GPa
  - shear modulus 40.0 GPa
Joint properties

- Joint spacing
  - normal stiffness: $2 \times 10^5$ MPa/m
  - shear stiffness: $2 \times 10^5$ MPa/m
  - permeability factor: 300 m · sec/kg
  - residual hydraulic aperture: $2 \times 10^{-5}$ m
  - aperture at zero normal stress: $1 \times 10^{-4}$ m
Initial Setting

;----- Pressurized borehole, biaxial stress field

config fluid

set flow clear incompressible off

• flow is assumed to be incompressible
• set flow clear: turns off all fluid flow settings
• incompressible on/off: turns on the incompressible fluid setting

round 1E-3

• All blocks in UDEC have “rounded” corners
• Prevents blocks from hanging up at sharp corners; block hang-up causes erroneous stress concentrations to occur in the model
• round d: d is the rounding distance.
• The default value is d = 0.5
• The rounding length is the same for all blocks in a model
Example of Rounded Blocks

Definition of rounded corners in UDEC

\[ d = r \]

\[ d \gg r \]
Initial Setting

round 1E-3

dedge 2E-3

- *edge emin*: The minimum block edge is set to emin.
- The default is twice the rounding length.
- emin must be greater than or equal to twice the rounding length
- *edge* must be specified prior to the *block* command
Geometry Setting, Example

```
block 0 0 0 5 10 5 10 0
jset angle 10 spacing 1.5 origin 2,0
jset angle 100 spacing 2.5 origin 0,0
```

- **origin x0,y0**: coordinates (global axis) of the start of one joint trace. A joint will be generated starting at (x0,y0); additional joints will be generated to fill the region.
Geometry Setting

- block -10,-10 -10,10 10,10 10,-10
- jset angle 10 spacing 2.5 origin 0,0
- jset angle 80 spacing 2.5 origin 0,0
Geometry Setting

arc (0,0) (0.2,0) 360 8 joint

delete range annulus (0,0) 0 0.2

\[ \text{annulus } x_c \ y_c \ r_1 \ r_2 \]

\text{annular range with center } (x_c, y_c) \text{ and radii } r_1 \text{ and } r_2
Mesh Setting

gen edge 5.0

**GENERATE**

keyword `value <...> <range...>`

All blocks with centroids lying within the optional `range` (see Section 1.1.3) are discretized into deformable **triangular finite-difference zones**. If no range is given, then all blocks will be discretized.

Zone generation can be performed automatically or manually. For automatic generation, one of the following keywords *must* be specified:

**edge**

`edmax` automatic generation of zones for an arbitrarily shaped block. The parameter `edmax` defines the maximum edge length of the triangular zones.
Mesh Setting

gen edge 5.0 vs gen edge 1
Block Property

group zone 'rock'

zone model elastic density 2.5E-3 bulk 6.667E4 shear 4E4 range group 'rock'
Joint Property

group joint 'jointA'

joint model area jks 2E5 jkn 2E5 jfriction 30 jperm 3E8 &
ares 2E-5 azero 1E-4 range group 'jointA'

The **model** keyword assigns constitutive models via the following names.

<table>
<thead>
<tr>
<th>model</th>
<th>keyword</th>
</tr>
</thead>
<tbody>
<tr>
<td>area</td>
<td>Coulomb slip (area contact)</td>
</tr>
<tr>
<td>bb</td>
<td>Barton-Bandis joint model (optional model)</td>
</tr>
<tr>
<td>cy</td>
<td>continuously yielding model</td>
</tr>
<tr>
<td>point</td>
<td>Coulomb slip (point contact)</td>
</tr>
<tr>
<td>residual</td>
<td>Coulomb slip with residual strength (area contact)</td>
</tr>
</tbody>
</table>

The following names are available.
Joint Property

Joint model `area` jks 2E5 jkn 2E5 `friction` 30 `perm` 3E8 &
`ares` 2E-5 `azero` 1E-4 range group 'jointA'

Point contact  
Area contact
Joint Property

Joint model area jks 2E5 jkn 2E5 jfriction 30 jperm 3E8 &
ares 2E-5 azero 1E-4 range group 'jointA'

Coulomb slip (area contact)

(1) **ares**  residual aperture at high stress
(2) **azer0**  aperture for zero normal stress
(3) **empb**  empirical multiplier for fluid flow law
(4) **expa**  exponent for joint hydraulic aperture
(5) **grdexp**  gradient exponent for fluid flow
(6) **jcohesion**  joint cohesion
(7) **jdilat0n**  joint dilation angle
(8) **jfriction**  joint friction angle
(9) **jkn**  joint normal stiffness
(10) **jks**  joint shear stiffness
(11) **jperm**  wetting fluid joint permeability
(12) **jtenso0n**  joint tensile strength
Joint Property

\[ \tau_f = c + \sigma \tan \phi \]

Joint model area jks 2E5 jkn 2E5 jfriction 30 jperm 3E8

ares 2E-5 azero 1E-4 range group 'jointA'
Joint Property

joint model area jks 2E5 jkn 2E5 jfriction 30 jperm 3E8
ares 2E-5 azero 1E-4 range group 'jointA'

; new contact default

set jcondf joint model area jks=2E5 jkn=2E5 jfriction=30 jperm=3E8 &
ares=2E-5 azero=1E-4

• It is important that a material model and properties be assigned to any new contacts that may be created during a model run. This can be accomplished with the SET jcondf command, which allows the specification of a joint model and properties for any new contacts created during the run.
Initial Conditions

insitu stress -25.0 ,0.0 , -20.0 pp 10
boundary xvelocity 0 range -10.1,-9.9 -10.1,10.1
boundary xvelocity 0 range 9.9,10.1 -10.1,10.1
boundary yvelocity 0 range -10.1,10.1 -10.1,-9.9
boundary yvelocity 0 range -10.1,10.1 9.9,10.1
solve ratio 1.0E-5 elastic
save bh0.sav

- insitu stress sxx sxy syy
- range: xl,xu yl,yu lower and upper limits for x and y

\[
\begin{align*}
\sigma_{xx} &= -25 \text{ MPa} \\
\sigma_{yy} &= -20 \text{ MPa} \\
p &= 10 \text{ MPa}
\end{align*}
\]
reset hist time
history ncyc 1
history pp -5.0,0.0
history pp 0.0,-5.0
history pp -9.0,0.0
history pp 0.0,-9.0
history pp 0.0,0.0
history flowtime
history unbvol

- **history ncyc**: A time history of selected variables is stored every ncyc timesteps
- **history pp x,y**: Location nearest to (x,y)
- **unbvol**: Maximum unbalanced fluid volume
Analysis Setting

set flow=on

boundary pp 10.0

well flow 0.001 atdomain (0,0)

- **well flow q atdomain (x,y)**
- **well**: Prescribes a fluid source in a domain
- A fluid source with flow rate q is placed at domain specified by x, y.
- If q is negative, a sink is assumed.
- q=10^-3 m3/sec
Analysis Setting

set maxmech=1000
set voltol=1.0E-4
set dtflow=0.5
cycle ftime 15.0
save bh1.sav

- **maxmech**: Maximum number of mechanical relaxation steps performed within a fluid cycle
- **voltol**: Mechanical stepping within a fluid cycle is stopped when the maximum ratio of unbalanced fluid volume to domain volume is below $V$ (default $V=0.001$)
- **dtflow**: Fluid timestep is prescribed (required with SET flow incompressible)
- **cycle**: Executes run
- **ftime**: If SET flow incompressible is invoked, fluid steps are executed. $f$ is the flow duration time.
Output: Pore Pressure

Fluid flow > pp
Given the difference in in-situ stresses, the dominant flow is in the horizontal joint set:

\[ \sigma_{xx} = -25 \text{ MPa} \]
\[ \sigma_{yy} = -20 \text{ MPa} \]
\[ p = 10 \text{ MPa} \]
Output: Fluid Velocity

Fluid flow > Bargraphs > Fvel
Output: Fluid Directions
Output: Joint Hydraulic Aperture
Output: History Monitoring, Pore Pressure

- $\sigma_{xx} = -25$ MPa
- $\sigma_{yy} = -20$ MPa
- $p = 10$ MPa
- $ftime = 15.0$

history $\text{pp} -5.0,0.0$
history $\text{pp} 0.0,-5.0$
history $\text{pp} -9.0,0.0$
history $\text{pp} 0.0,-9.0$
history $\text{pp} 0.0,0.0$
Outline

- Numerical Analysis Methodology
- Wellbore Stability Analysis
- Flow from Boreholes in Biaxial Stress Fields
- Hydraulic Fracturing Simulation
Hydraulic Fracturing Simulation

UDEC
Hydraulic Fracturing Simulation

- A uniform fluid pressure is applied inside a planar crack
- The displacements of the crack surface is measured
- The medium is assumed to be elastic.
- The crack is straight with a length of 21.6 m
- **UDEC sign conventions:**
  - tension and extension are positive for the rock matrix
  - Joint opening is positive
  - joint normal stress and fluid pressure are positive in compression

<table>
<thead>
<tr>
<th>E  [MPa]</th>
<th>ν</th>
<th>σ_{yy} [MPa]</th>
<th>σ_{xx} [MPa]</th>
<th>a [m]</th>
<th>p [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>40,000</td>
<td>0.22</td>
<td>-15</td>
<td>-30</td>
<td>10.8</td>
<td>20</td>
</tr>
</tbody>
</table>
Model Parameters

new
;file hf_case1.dat
config fluid
title
Uniform pressure
def setup
_young = 40e3 ; MPa
_nu = 0.22
_syy = -15.
_sxx = -30.
_pp = 20.
_pp = 20.
_hl = 10.8 ; fracture half length ;

_ijkn = 3e4
_ares = 5e-5
_a0 = 5e-4
end
setup
Model Geometry

round 0.01


crack -24 0 24 0

gen edge 4.5

join_contact

join_contact off range -10.5 10.5 -1 1

CRACK table n <id = n> <join>

x1 y1 x2 y2 <range...> <id = n> <join>

A crack is created between points \((x_1, y_1)\) and \((x_2, y_2)\). CRACK can be used to create a discontinuous fracture in part of a block. Two cracks will be connected if their endpoint locations are within a distance of twice the rounding length from each other. All discontinuous cracks that do not link to form blocks are deleted when the GENERATE or CYCLE command is executed.
prop mat 1 dens 1e-3 ymod _young prat _nu
; --- crack ---
joint model resid jkn _jkn jks _jkn jperm 300 ares=ares
fluid dens 1e-3
insitu stress _sxx 0 _syy szz 0 pp 0 aperture 0.0
boundary stress _sxx 0 _syy
cycle 1

The following names are available.

- **area**: Coulomb slip (area contact)
- **bb**: Barton-Bandis joint model (optional model)
- **cy**: continuously yielding model
- **point**: Coulomb slip (point contact)
- **residual**: Coulomb slip with residual strength (area contact)

3. Stress Boundary

**stress**

- **sxxo sxyo syyo**

boundary stress parameters: xx-stress, xy-stress and yy-stress
Boundary Element Representation of the Far Field

; --- boundary element representation of the far field ---
be gen -24 24 -24 24
be mat 1
be fix 0 -24 -24 0
be stiff

BE keyword <keyword>

Boundary-element (limit 300) conditions are applied to the boundary domain generated by the keyword gen. The keywords and associated parameters are:

gen <xl xu yl yu>
<corner ic1 ic2>

generates a boundary-element domain along the outer boundary of a region described by xl to xu and yl to yu, or block corners ic1 and ic2. (The affected boundary runs clockwise from ic1 to ic2.)

mat n*

material number n assigned to far-field properties

fix xh yh xv yy

fixes a point (xh, yh) in the x-direction and a point (xv, yy) in the y-direction outside the model region (must precede the stiff keyword).

stiff

automatically generates stiffness matrix.
Analysis Setting and PP Boundary Conditions

set dscan 100000 ; turn off scan for new contacts
; to speed calculation

dscan n

sets frequency of domain check. The default is every three cycles.

; uniform pressure in fracture
pfix p \_pp range -10.5 10.5 -.1 .1
;
; keep zero pp in impermeable joints
pfix p=0 range -50 -10.5 -.1 .1
pfix p=0 range 10.5 50 -.1 .1
Monitoring Setting

hist ydis 0 0 ydis 2 0 ydis 4 0 ydis 6 0 ydis 8 0 ydis 10 0
Analysis Procedure

save hf_case1_ft.sav
set flow steady
set flow off
set caprat 100
solve force 0 ratio 1e-6
save hf_case1.sav
;rest hf_case1.sav

Fluid flow is turned off and the model is cycled to mechanical equilibrium

cap\text{ratio} \quad r

Maximum contact hydraulic aperture is limited to $r$ times the maximum residual aperture. (Default is 5.) Note that the maximum hydraulic aperture is a global variable based on the maximum residual aperture in the model. It does not vary for different contacts.

ratio \quad value

ratio limit for the mechanical calculation process. \text{ratio} is the default limit, and the default value is $10^{-5}$.

force \quad f

out-of-balance force limit. (default is $f = 100$)
Result Extraction (ydisp vs step)
Fracture Opening along the Fracture

The analytical solution for fracture opening, Parker 1981

\[ w = \frac{|\sigma_{yy} + p|}{E} \frac{4(1 - v^2)}{a^2 - x^2} \sqrt{a^2 - x^2} \]

a: Fracture half length
Displacement vectors at the end of the simulation
Other Outputs

- Vertical stress
- Horizontal stress
- Pore pressure
- Bargraphs > Aperture
- Contour Motion > Y-Displacement
- Contour Motion > X-Displacement
- Contour Motion > Displacement magnitude
Severe Condition

PP=10 MPa

PP=500 MPa
Summary

- Numerical Analysis Methodology
- Wellbore Stability Analysis
- Flow from Boreholes in Biaxial Stress Fields
- Hydraulic Fracturing Simulation
Thank you!

Any Questions?
Previous Workshops
“Numerical Modeling in Geotechnical Engineering” Workshop
Jan 2016

Attendees from:
AECOM
Leidos engineering Inc.
Southern Illinois University Edwardsville
“Numerical Modeling in Geotechnical Engineering” Workshop
Oct 2014

Attendees from:
Geotechnology, Inc.
Subsurface Constructors, Inc.
University of Missouri – Columbia
Missouri University of Science and Technology
Southern Illinois University Edwardsville
References

UDEC, Itasca Inc. 2016
FLAC3D, Itasca Inc. 2016
Steven F. Bartlett, Numerical Methods in Geotechnical Engineering, The University of Utah, 2012