Introduction to Computational Geotechnics

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“Numerical Modeling in Geotechnical Engineering” Workshop
Jan 2016

Attendees from:
AECOM
Leidos engineering Inc.
Southern Illinois University Edwardsville
“Numerical Modeling in Geotechnical Engineering” Workshop
Oct 2014

Attendees from:
Geotechnology, Inc.
Subsurface Constructors, Inc.
University of Missouri – Columbia
Missouri University of Science and Technology
Southern Illinois University Edwardsville
References


Steven F. Bartlett, Numerical Methods in Geotechnical Engineering, The University of Utah, 2012

Plaxis Manual, 2007
1. Introduction to Computational Geotechnics
   1. Numerical modeling approach
   2. Idealized field conditions to numerical modeling
   3. Algorithm of numerical modeling
2. Commercial geotechnical programs
   1. Programs developed by Itasca, Inc.
   2. Programs developed by Plaxis
   3. Programs developed by Geo-Slope International Ltd.
   4. Other products
3. Theoretical considerations
   1. Numerical methods
   2. Strength of material
   3. Constitutive models
Outline

4. Numerical modeling in FLAC
   Part I
   1. Introductory of modeling in FLAC
   2. Grid generation
   3. Geometry changes
   4. Shallow foundation
   Part II
   5. Stone column
   6. Slope stability
   7. Soil nailing
   8. Seismic considerations
5. Numerical modeling in Plaxis
   • Shallow foundations
Customized Training Courses

Numerical modeling in FLAC, FLAC3D, Plaxis
- Shallow and deep foundation (pile, stone columns)
- Tunneling
- Retaining wall
- Slope stability
- Soil reinforcement systems
- Soil nailing
- Soil anchoring
- Micropiles
- MSE walls
- Levees
- Dynamic analysis
- Flow analysis

If you would like to schedule Siavash for a presentation or workshop, please contact zamiran@siu.edu or visit www.zamiran.net
Chapter 1

Introduction to Computational Geotechnics
1-1 Numerical Modeling Approach
Numerical Modeling

![Diagram showing the relationship between Genesis/Geology, Ground Profile, Soil Behaviour, and Modeling.]

- Genesis/Geology
- Ground Profile
  - Site investigation, ground description
  - Empiricism, precedent, experience, risk management
- Soil Behaviour
  - Lab/field testing, observation, measurement
- Modeling
  - Idealization followed by evaluation. Conceptual or physical modeling, analytical modeling
Numerical Modeling Procedure

Observe
Measure
Explain
Verify
Results
Simple Definition of Modeling

![Diagram showing the relationship between Physical System, Mathematical System, and Numerical System.](image)
Application of Computational Geotechnics

- Interpretation
- Design
- Prediction

Figure 3. Predicted and measured deflections of a tie-back wall (Carter et al. 2000).
1-2 Idealized Field Conditions to Numerical Modeling
Idealize Field Conditions to Numerical Modeling

3D modeling

2D modeling

- Plain strain
  - No strain in the z direction
  - Structure or feature is relatively long

- Axi-symmetry
Plan Strain vs. Axi-symmetry
Plain Strain Numerical Modeling Examples

Deformation analysis of slopes

Deformation analysis of tunnels
Plain Strain Numerical Modeling Examples

Dynamic analysis

MSE walls

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Plain Strain Numerical Modeling Examples

Retaining wall

Embankment dam

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Plain Strain Numerical Modeling Examples

Strip footing

Roadway embankment
Axisymmetric Conditions
Axisymmetrical Numerical Modeling Examples

Circular footing

Single pile
Axisymmetrical Numerical Modeling Examples

Flow to an injection and/or pumping well

Point load on soil
1-3 Algorithm of Numerical Modeling
Numerical Modeling Steps

Selection of representative cross-section
  Idealize the field conditions into a design X-section
  Plane strain vs. axisymmetrical models
Choice of numerical method and program
Defining the geometry
Assign constitutive model e.g. elastic, Mohr-Coulomb, etc.
Assign material properties
Generate grid/mesh for the domain
Assign boundary/loading conditions
Solve for initial condition
Problem alterations
Run the model
Obtain results
Interpret of results
Numerical Flowchart

MODEL SETUP
1. Generate grid, deform to desired shape
2. Define constitutive behavior and material properties
3. Specify boundary and initial conditions

Start

Step to equilibrium state

Examine the model response

More tests needed

PERFORM ALTERATIONS for example,
- Excavate material
- Change boundary conditions

Step to solution

Examine the model response

Acceptable result

Parameter study needed

Yes

No

End

Results unsatisfactory

Model makes sense

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Chapter 2
Commercial Geotechnical Programs
Overview

- Programs Developed by Itasca, Inc.
- Programs Developed by Plaxis
- Programs Developed by Geo-Slope International Ltd.
- Programs Developed by Rocscience
- Programs Developed by Midas Technology, Inc.
2-1 Programs Developed by Itasca, Inc.
Itasca Consulting Group, Inc.

- Engineering consulting and software firm
- Based on Minneapolis, MN
- Areas of concentration: mining, civil engineering, oil & gas, manufacturing and power generation
- Since 1981
- Products:
  - FLAC
  - FLAC3D
  - Flac/ Slope
  - PFC
  - 3DEC
  - UDEC
FLAC/FLAC3D

- Large-strain simulation of continua
- Groundwater flow, with full coupling to mechanical calculation
- Structural elements
- Thermal and creep calculations
- Dynamic analysis
- Two-phase fluid flow model
- User-defined constitutive models written in C++
- Built-in language (FISH) to add user-defined features (e.g., new constitutive models, new variables or new commands)

**FLAC Slope**

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FLAC- Download a Demo Version

You are not logged in.

I have an account  I want to create an account

szamira –Sha1

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FLAC
Terminology
UDEC/3DEC

- Stability analysis of jointed rock slopes
- Deep underground excavations
- Blasting effects
- Ground support reinforcement
- Underground construction
- Fluid-pressurized tunnels
- Dams and dam foundations
- Fluid flow through jointed rock (hydraulic fracturing)
- Earthquake engineering
UDEC- Dams and dam foundations
UDEC/3DEC - Tunnels
PFC2D/3D

- Particle Flow Code
- Thermal-mechanical coupling
- Add new physics using C++
- Available fluid dynamics add-on
2-2 Programs Developed by Plaxis
Plaxis

- Finite element method
- 2-Dimensional and 3-Dimensional analysis
- Groundwater flow
- Heat flow
- Dynamic analysis
- Based on Delft, The Netherlands
- Products:
  - Plaxis 2D
  - Plaxis 3D
  - 3D Plaxiflow
  - 2D Plaxflow
Plaxis2D
Plaxis2D
Plaxis3D
2-3 Programs Developed by GEO-SLOPE International Ltd.
Geo-Slope Products

- SLOPE/W for slope stability
- SEEP/W for groundwater seepage
- SIGMA/W for stress-deformation
- QUAKE/W for dynamic earthquake
- TEMP/W for geothermal
- CTRAN/W for contaminant transport
- AIR/W for air flow

- Based on Alberta, Canada
- 2-Dimensional program
Slope/W

Published FOS = 1.04
(After Duncan, Wright and Wong, 1990)
Sigma/W
Seep/W
2-4 Other Products
Rocscience

- Slide:
  - Slope stability analysis software
  - with built-in finite element groundwater seepage analysis
- RS:
  - 2D finite element program for soil and rock applications
Slide
Midas Technology, Inc

Midas GTS

- Finite element analysis software
- Deep Foundations
- Excavations
- Complex Tunnel Systems
- Seepage Analysis
- Consolidation Analysis
- Embankment Design
- Dynamic and slope stability analysis
Midas GTS
Chapter 3

Theoretical Considerations
3-1 Numerical Methods
Numerical Methods

Continuum modelling
Discontinuum modelling
Limit equilibrium
Hybrid/coupled modelling
Continuum Modelling

Finite element
- Plaxis
- Plaxis3D
- SIGMA/W
- Midas GTS
- RS

Finite difference
- FLAC
- FLAC3D

Discontinuum modelling

Distinct (discrete) element method:
Joints are treated as boundary conditions. Deformable blocks are discretized into internal constant-strain elements
- UDEC
- 3DEC

Particle flow codes
- PFC2D
- PFC3D

Hybrid/coupled modelling
- FLAC
- FLAC3D
- UDEC
- 3DEC

Limit equilibrium
- Slope/W
- Slide
Finite Element Method

Introduced from mechanical and structural analysis of beam, columns, frames, etc.

Developed into continuous media => soil

Division of domain geometry => finite element mesh

Matrix operations for formulation

Stiffness matrix generated

Adjustment of field variables is made => error term is minimized (energy)
Finite Difference Method

Oldest & simplest technique
No matrix operations
Field variables
  □ Stress or pressure
  □ Displacement
  □ Velocity
Solution is done by time stepping (small interval of time)
Each time step: grid values are updated
Good method for:
  □ Dynamic analysis
  □ Large deformation analysis
Finite Difference Calculation Cycle

- **Equilibrium Equation** (Equation of Motion)
- New velocities and displacements
- **Stress / Strain Relation** ( Constitutive Equation)
- New stresses or forces
FD & FE Typical Meshing System
Element vs. Grid

Element (FE)  

Grid, Zone (FD)
Limit Equilibrium

- Slide
- SLOPE/W

✓ Safety factor
3-2 Strength of Material
Constitutive Relationships I

Elastic
- Linear elasticity equations
- Hooke's law

Viscoelastic
- Behave elastically
- Also has damping (when the stress is applied and removed)

Elasto-plastic
- Applied stress is less than a yield value: elastic
- More: plastic
Constitutive Relationships II

Elastic - Plastic Behavior

Viscoelastic Behavior
Normal and Shear Stresses

\[ \overline{\sigma} = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} \]

- Normal stress in the x direction
- Normal stress in the y direction
- Normal stress in the z direction
- Shear stress on the xy plane
- Shear stress on the yz plane
- Shear stress on the zx plane

Recall that:

\[ \tau_{xy} = \tau_{yx} \]
\[ \tau_{yz} = \tau_{zy} \]
\[ \tau_{zx} = \tau_{xz} \]
Normal and Shear Strain

\[ \varepsilon_x = \frac{\partial u}{\partial x} \quad \text{Axial strain in the x-direction} \]

\[ \varepsilon_y = \frac{\partial v}{\partial y} \quad \text{Axial strain in the y-direction} \]

\[ \varepsilon_z = \frac{\partial w}{\partial z} \quad \text{Axial strain in the z-direction} \]

\[ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad \text{Shear strain in the x-y plane} \]

\[ \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \quad \text{Shear strain in the y-z plane} \]

\[ \gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \quad \text{Shear strain in the z-x plane} \]
Strain - Displacement

\[ \mathbf{\varepsilon} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \]

Axial strain in the x direction
Axial strain in the y direction
Axial strain in the z direction
Shear strain in the xy plane
Shear strain in the yz plane
Shear strain in the zx plane

\[ \mathbf{u} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \]

Displacement in the x direction
Displacement in the y direction
Displacement the z direction
Hooke’s Law

Stains from Hooke’s Law

\[
\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E}
\]
\[
\varepsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E} - \nu \frac{\sigma_z}{E}
\]
\[
\varepsilon_z = \frac{\sigma_z}{E} - \nu \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E}
\]
\[
\gamma_{xy} = \frac{\tau_{xy}}{G}
\]
\[
\gamma_{yz} = \frac{\tau_{yz}}{G}
\]
\[
\gamma_{zx} = \frac{\tau_{zx}}{G}
\]

and

\[
\sigma_x = \frac{E}{(1+\nu)(1-2\nu)} \left[ (1-\nu)\varepsilon_x + \nu(\varepsilon_y + \varepsilon_z) \right]
\]
\[
\sigma_y = \frac{E}{(1+\nu)(1-2\nu)} \left[ (1-\nu)\varepsilon_y + \nu(\varepsilon_x + \varepsilon_z) \right]
\]
\[
\sigma_z = \frac{E}{(1+\nu)(1-2\nu)} \left[ (1-\nu)\varepsilon_z + \nu(\varepsilon_x + \varepsilon_y) \right]
\]

\[
E = \text{elastic modulus}
\]
\[
\nu = \text{poisson’s ratio}
\]
\[
G = \text{shear modulus}
\]

\[
G = \frac{E}{2(1+\nu)}
\]

E ≡ \text{tensile stress} \quad \frac{\sigma}{\varepsilon}

G \equiv \frac{\tau_{xy}}{\gamma_{xy}}
Bulk Modulus

\[ K = -V \frac{\partial P}{\partial V} \]

P: pressure
V: volume
\( \partial P/\partial V \): partial derivative of pressure with respect to volume
Elastic Correlations

<table>
<thead>
<tr>
<th></th>
<th>((G, \nu))</th>
<th>((E, \nu))</th>
<th>((K, \nu))</th>
<th>((K, E))</th>
<th>((M, G))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K)</td>
<td>(\frac{2G(1+\nu)}{3(1-2\nu)})</td>
<td>(\frac{E}{3(1-2\nu)})</td>
<td>(3K(1-2\nu))</td>
<td>(\frac{3K(3K-E)}{9K-E})</td>
<td>(M - \frac{4G}{3})</td>
</tr>
<tr>
<td>(E)</td>
<td>(2G(1+\nu))</td>
<td>(\frac{E\nu}{(1+\nu)(1-2\nu)})</td>
<td>(3K(1-2\nu))</td>
<td>(\frac{3KE}{9K-E})</td>
<td>(\frac{G(3M-4G)}{M-G})</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>(\frac{2G\nu}{1-2\nu})</td>
<td>(\frac{E\nu}{(1+\nu)(1-2\nu)})</td>
<td>(\frac{3K\nu}{1+\nu})</td>
<td>(\frac{3K(3K-E)}{9K-E})</td>
<td>(M - 2G)</td>
</tr>
<tr>
<td>(G)</td>
<td>(\frac{E}{2(1+\nu)})</td>
<td>(\frac{3K(1-2\nu)}{2(1+\nu)})</td>
<td>(\frac{3KE}{9K-E})</td>
<td>(\frac{3K-E}{6K})</td>
<td>(\frac{M-2G}{2M-2G})</td>
</tr>
<tr>
<td>(\nu)</td>
<td>(\frac{2G(1-\nu)}{1-2\nu})</td>
<td>(\frac{E(1-\nu)}{(1+\nu)(1-2\nu)})</td>
<td>(\frac{3K(1-\nu)}{1+\nu})</td>
<td>(\frac{3K(3K+E)}{9K-E})</td>
<td>(\frac{M-2G}{2M-2G})</td>
</tr>
</tbody>
</table>
In a continuous solid body:

\[ \rho \frac{\partial \ddot{u}_i}{\partial t} = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho g_i \]

where \( \rho \) = mass density; \( t \) = time; \( x_i \) = components of coordinate vector; \( g_i \) = components of gravitational acceleration (body forces); and \( \sigma_{ij} \) = components of stress tensor.
3-3 Constitutive Models
Elastic Model

The simplest representation of material behavior
Homogeneous
Isotropic
Continuous materials
Material that exhibit linear stress-strain behavior with no hysteresis on unloading

- Bulk modulus
- Shear modulus
- Modulus of elasticity
- Poisson's ratio
Mohr-Coulomb Model (Elastoplastic)

Conventional model used to represent shear failure in soils and rocks

Elastic portion
- Bulk modulus
- Shear modulus
- Modulus of elasticity
- Poisson's ratio

Plastic portion
- Cohesion
- Friction angle
- Dilation angle
- Tension

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## Other Constitutive Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drucker-Prager</td>
<td>Soft clays with low friction angles</td>
</tr>
<tr>
<td>Ubiquitous-joint</td>
<td>Developed through Mohr-Coulomb solid, anisotropic</td>
</tr>
<tr>
<td>Strain-hardening/softening</td>
<td>Nonlinear material softening and hardening behavior</td>
</tr>
<tr>
<td>Modified Cam-clay model</td>
<td>Soft clay</td>
</tr>
<tr>
<td>Hoek-Brown model</td>
<td>Intact rock and rock masses</td>
</tr>
</tbody>
</table>
Chapter 4 – Part I

Numerical Modeling in FLAC – Part I
4. Numerical modeling in FLAC
   Part I
   1. Introductory of modeling in FLAC
   2. Grid generation
   3. Geometry changes
   4. Shallow foundation
   Part II
   5. Stone column
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   8. Seismic considerations
Constitutive Relation

strain rate is derived from velocity gradient as follows

\[ \dot{e}_{ij} = \frac{1}{2} \left[ \frac{\partial \dot{u}_i}{\partial x_j} + \frac{\partial \dot{u}_j}{\partial x_i} \right] \]

where \( \dot{e}_{ij} \) = strain-rate components; and
\( \dot{u}_i \) = velocity components.

Mechanical constitutive laws are of the form

\[ \sigma_{ij} := M(\sigma_{ij}, \dot{e}_{ij}, \kappa) \]

where \( M(\ ) \) is the functional form of the constitutive law;
\( \kappa \) is a history parameter(s) which may or may not be present, depending on the particular law; and
\( := \) means “replaced by.”
Constitutive Model

\[ \sigma_{ij} := M(\sigma_{ij}, \dot{e}_{ij}, \kappa) \]

The simplest example of a constitutive law: isotropic elasticity:

\[ \sigma_{ij} := \sigma_{ij} + \left\{ \delta_{ij} \left( K - \frac{2}{3} G \right) \dot{e}_{kk} + 2G \dot{e}_{ij} \right\} \Delta t \]

where \( \delta_{ij} \) is the Kronecker delta;
\( \Delta t \) = timestep; and
\( G, K \) = shear and bulk modulus, respectively.
Finite Difference Zones

Figure 1.3  
(a) Overlaid quadrilateral elements used in FLAC
(b) Typical triangular element with velocity vectors
(c) Nodal force vector
Zone and Gridpoint

```
+-----+-----+-----+-----+-----+-----+-----+-----+
|     |     | zone|     |     |     |     |     |
+-----+-----+-----+-----+-----+-----+-----+-----+
| 1   | 2    | 3   | 4   | i-1 | i   | i+1 |
| 1   | 2    | 3   | 4   | i-1 | i   | i+1 |
+-----+-----+-----+-----+-----+-----+-----+-----+
```

velocities, displacements

stresses

gridpoint
Finite Difference Grid with 400 Zones
Zone Numbers

![Finite difference grid diagram with Zone Numbers](image-url)

Legend:
- 30-Dec-10 15:57
- step 0
- -1.000E+00 < x < 7.000E+00
- -1.000E+00 < y < 7.000E+00

Grid plot:
- 0 to 2E0

Zone Numbers:
- 1.1, 2.1, 3.1, 4.1, 5.1, 6.1, 7.1
- 1.2, 2.2, 3.2, 4.2, 5.2, 6.2, 7.2
- 1.3, 2.3, 3.3, 4.3, 5.3, 6.3, 7.3
- 1.4, 2.4, 3.4, 4.4, 5.4, 6.4, 7.4
- 1.5, 2.5, 3.5, 4.5, 5.5, 6.5, 7.5
- 1.6, 2.6, 3.6, 4.6, 5.6, 6.6, 7.6
- 1.7, 2.7, 3.7, 4.7, 5.7, 6.7, 7.7

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Grid Point Numbers
Boundary Condition

Boundary fixed in x direction

Typical boundary conditions
- Fixed in x direction
- Fixed in y direction
- Fixed in both directions
- Free in x and y directions (no boundary assigned)
Initial Conditions

Initial Conditions that are generally considered:
- Initial shear stresses
- Groundwater conditions
  - Hydrostatic water table
  - Flow gradient (non-steady state)
- For dynamic problems
  - Acceleration, velocity or stress time history

Initial groundwater conditions

Effective vertical stress contours
Boundary Conditions

- Fixed (X or Y) or both (B)
- Free

X means fixed in x direction
B means fixed in both directions
Applied Condition

- Velocity or displacement
- Stress or force

Yellow line with circle means force, velocity or stress has been applied to this surface.
Grids

- Tunnel
- Rock Slope with groundwater
- Slope or Embankment
- Concrete Diaphragm Wall
General solution procedure:

Start:
COMMAND keyword value . . . <keyword value . . . > . . .

; comments

grid icol jrow

grid 10 10
model elastic

grid 20,20
model elas
gen 0.5 0,20 20,20 5,5 i=11
gen same same 20,0 5,0 i=11,21

grid 20,20
me
gen 0,0 0,100 100,100 100,0 rat 1.25 1.25
Creating a circular hole in a grid

new
grid 20,20
me
gen circle 10,10 5
model null region 10,10
model null region 10,10

Moving gridpoints with the INITIAL command

new
grid 5 5
model elastic
gen 0,0 0,10 10,10 10,0
ini x=-2 i=1 j=6
ini x=12 i=6
BAD GEOMETRY

(1) the area of the quadrilateral must be positive; and
(2) each member of at least one pair of triangular subzones which comprise the quadrilateral must have an area greater than 20% of the total quadrilateral area.

Figure 2.40 Acceptable and unacceptable zone deformations
Assigning Material Models

Elastic Model
MODEL elastic and MODEL mohr-coul require that material properties be assigned via the PROPERTY command. For the elastic model, the required properties are
(1) density;
(2) bulk modulus; and
(3) shear modulus.

\[
K = \frac{E}{3(1-2\nu)}
\]

\[
G = \frac{E}{2(1+\nu)}
\]

\[
E = \frac{9KG}{3K+G}
\]

\[
\nu = \frac{3K-2G}{2(3K+G)}
\]
Mohr-Coulomb plasticity model

(1) density;
(2) bulk modulus;
(3) shear modulus;
(4) friction angle;
(5) cohesion;
(6) dilation angle; and
(7) tensile strength.

grid 10,10
model elas j=6,10
prop den=2000 bulk=1e8 shear=.3e8 j=6,10
model mohr j=1,5
prop den=2500 bulk=1.5e8 shear=.6e8 j=1,5
prop fric=30 coh=5e6 ten=8.66e6 j=1,5
## Applying Boundary and Initial Conditions

<table>
<thead>
<tr>
<th>Command</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>APPLY</strong></td>
<td></td>
</tr>
<tr>
<td>pressure</td>
<td>mechanical pressure (<em>not</em> pore pressure) applied at boundary</td>
</tr>
<tr>
<td>sxx</td>
<td>$xx$-component of total stress tensor applied at boundary</td>
</tr>
<tr>
<td>sxy</td>
<td>$xy$-component of total stress tensor applied at boundary</td>
</tr>
<tr>
<td>syy</td>
<td>$yy$-component of total stress tensor applied at boundary</td>
</tr>
<tr>
<td>xforce</td>
<td>$x$-component of force applied at boundary gridpoints</td>
</tr>
<tr>
<td>yforce</td>
<td>$y$-component of force applied at boundary gridpoints</td>
</tr>
<tr>
<td>xvelocity</td>
<td>$x$-velocity applied at boundary gridpoints</td>
</tr>
<tr>
<td>yvelocity</td>
<td>$y$-velocity applied at boundary gridpoints</td>
</tr>
<tr>
<td><strong>FIX</strong></td>
<td></td>
</tr>
<tr>
<td>pp</td>
<td>pore pressure fixed at boundary gridpoints</td>
</tr>
<tr>
<td>x</td>
<td>$x$-velocity fixed at boundary gridpoints</td>
</tr>
<tr>
<td>y</td>
<td>$y$-velocity fixed at boundary gridpoints</td>
</tr>
</tbody>
</table>
Sign Conventions
DIRECT STRESS – Positive stresses indicate tension; negative stresses indicate compression.

SHEAR STRESS
Figure 2.44 Distortion associated with positive and negative shear strain

Figure 2.45 Mechanical pressure: (a) positive; (b) negative
**PORE PRESSURE** – Fluid pore pressure is positive in compression. Negative pore pressure indicates fluid tension.

**GRAVITY** – Positive gravity will pull the mass of a body downward (in the negative y-direction).

Negative gravity will pull the mass of a body upward.

---

### Table 2.5 Systems of units – mechanical parameters

<table>
<thead>
<tr>
<th></th>
<th>SI</th>
<th>Imperial</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length</strong></td>
<td>m</td>
<td>m</td>
</tr>
<tr>
<td><strong>Density</strong></td>
<td>kg/m³; 10³ kg/m³</td>
<td>m³</td>
</tr>
<tr>
<td><strong>Force</strong></td>
<td>N</td>
<td>10⁶ kg/m³</td>
</tr>
<tr>
<td><strong>Stress</strong></td>
<td>Pa</td>
<td>10⁶ g/cm³</td>
</tr>
<tr>
<td><strong>Gravity</strong></td>
<td>m/sec²</td>
<td>MPa</td>
</tr>
<tr>
<td><strong>Stiffness</strong></td>
<td>Pa/m</td>
<td>kPa/m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MPa/m</td>
</tr>
</tbody>
</table>

* Stiffness refers to normal and shear stiffnesses at interfaces.

where

1 bar = 10⁶ dynes/cm² = 10⁵ N/m² = 10⁵ Pa;

1 atm = 1.013 bars = 14.7 psi = 2116 lbf/ft² = 1.01325 × 10⁵ Pa;

1 slug = 1 lbf - s²/ft = 14.59 kg;

1 snail = 1 lbf - s²/in; and

1 gravity = 9.81 m/s² = 981 cm/s² = 32.17 ft/s².
<table>
<thead>
<tr>
<th>Parameter</th>
<th>SI</th>
<th>Imperial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water Bulk Modulus</td>
<td>Pa</td>
<td>lbf/ft²</td>
</tr>
<tr>
<td>Water Density</td>
<td>kg/m³</td>
<td>slugs/ft³</td>
</tr>
<tr>
<td>Permeability</td>
<td>m³/sec/kg</td>
<td>ft³/sec/slug</td>
</tr>
<tr>
<td>Intrinsic Permeability</td>
<td>m²</td>
<td>ft²</td>
</tr>
<tr>
<td>Hydraulic Conductivity</td>
<td>m/sec</td>
<td>ft/sec</td>
</tr>
</tbody>
</table>

**Table 2.6 Systems of units — groundwater flow parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SI</th>
<th>Imperial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water Bulk Modulus</td>
<td>Pa</td>
<td>lbf/ft²</td>
</tr>
<tr>
<td>Water Density</td>
<td>kg/m³</td>
<td>slugs/ft³</td>
</tr>
<tr>
<td>Permeability</td>
<td>m³/sec/kg</td>
<td>ft³/sec/slug</td>
</tr>
<tr>
<td>Intrinsic Permeability</td>
<td>m²</td>
<td>ft²</td>
</tr>
<tr>
<td>Hydraulic Conductivity</td>
<td>m/sec</td>
<td>ft/sec</td>
</tr>
</tbody>
</table>

**NOTE:** 
FLAC permeability is the *mobility coefficient* (coefficient of pore pressure term in Darcy’s law).

FLAC permeability in SI units = intrinsic permeability in cm² × 9.9 × 10⁻²
FLAC permeability in Imperial units = hydraulic conductivity in cm/sec × 1.02 × 10⁻⁶
<table>
<thead>
<tr>
<th>Property</th>
<th>Unit</th>
<th>SI</th>
<th>Imperial</th>
</tr>
</thead>
<tbody>
<tr>
<td>area</td>
<td>length$^2$</td>
<td>m$^2$</td>
<td>ft$^2$</td>
</tr>
<tr>
<td>axial or shear stiffness</td>
<td>force/length</td>
<td>N/m</td>
<td>lbf/ft</td>
</tr>
<tr>
<td>bond stiffness</td>
<td>force/length/length</td>
<td>N/m/m</td>
<td>lbf/ft/ft</td>
</tr>
<tr>
<td>bond strength</td>
<td>force/length</td>
<td>N/m</td>
<td>lbf/ft</td>
</tr>
<tr>
<td>exposed perimeter</td>
<td>length</td>
<td>m</td>
<td>ft</td>
</tr>
<tr>
<td>moment of inertia</td>
<td>length$^4$</td>
<td>m$^4$</td>
<td>ft$^4$</td>
</tr>
<tr>
<td>plastic moment</td>
<td>force-length</td>
<td>N-m</td>
<td>ft-lbf</td>
</tr>
<tr>
<td>yield strength</td>
<td>force</td>
<td>N</td>
<td>lbf</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>stress</td>
<td>kPa</td>
<td>lbf/ft$^2$</td>
</tr>
</tbody>
</table>

where $1 \text{ bar} = 10^6 \text{ dynes/cm}^2 = 10^5 \text{ N/m}^2 = 10^5 \text{ Pa}$. 
;Example
grid 10 10
model
fix x i=1
fix x i=11
fix y j=1
app press = 10 j=11
ini sxx=-10 syy=-10

hist unbal

hist xvel i=5 j=5
hist ydisp i=5 j=11
grid 10 10
model
prop d=1800 bulk=1e8 shear =.3e8
fix x i=1
fix x i=11
fix y j=1
app pres=1e6 j=11
hist unbal
hist ydisp i=5 j=11
ini sxx=-1e6 syy=-1e6 szz=-1e6
set gravity=9.81
step 900
Performing Alterations
FLAC allows model conditions to be changed at any point in the solution process. These changes may be of the following forms.
• excavation of material
• addition or deletion of gridpoint loads or pressures
• change of material model or properties for any zone
• fix or free velocities for any gridpoint

;Example
grid 10,10
model elastic
gen circle 5,5 2
plot hold grid
gen adjust
plot hold grid
prop s=.3e8 b=1e8 d=1600
set grav=9.81

fix x i=1
fix x i=11
fix y j=1
solve
ini sx 0.0 syy 0.0 szz 0.0 region 5,5
prop s 0.3e5 b 1e5 d 1.6 region 5,5
;mod null region 5,5
solve
**Excavate and fill in stages**

grid 10,10  
me  
prop s=5.7e9 b=11.1e9 d=2000  
fix x i=1  
fix y j=1  
fix x i=11  
apply syy -20e6 j=11  
ini sxx -30e6 syy -20e6 szz -20e6  
his unbal  
his xdis i=4 j=5  
solve  
mod null i 4,7 j 3,6  
solve  

mod mohr i 4,7 j 3,6  
prop s=.3e8 b=1e8 fric=30 i=4,7 j=3,6  
mod null i=1,3 j=3,6  
mod null i=8,10 j=3,6  
ini xd=0 yd=0  
his reset  
his unbal  
his xdis i=4 j=5  
step 1000
### Saving/Restoring Problem State

- `save file.sav`
- `rest file.sav`
- `save fill1.sav`

<table>
<thead>
<tr>
<th>Function</th>
<th>Command</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Grid Generation</strong></td>
<td>GRID GENERATE INITIAL</td>
</tr>
<tr>
<td><strong>Boundary/Initial Conditions</strong></td>
<td>APPLY FIX INITIAL</td>
</tr>
<tr>
<td><strong>Material Model &amp; Properties</strong></td>
<td>MODEL PROPERTY</td>
</tr>
<tr>
<td><strong>Initial Equilibrium</strong></td>
<td>STEP SOLVE SET gravity</td>
</tr>
<tr>
<td>(with gravity)</td>
<td></td>
</tr>
<tr>
<td><strong>Perform Alterations</strong></td>
<td>MODEL PROPERTY APPLY FIX FREE</td>
</tr>
<tr>
<td><strong>Save/Restore Problem State</strong></td>
<td>SAVE RESTORE</td>
</tr>
</tbody>
</table>
Example 1: Shallow Footing

;EXAMPLE 1
config
grid 25 10
;-------------------CONST MODEL-----------------
model elastic i=1,25 j=1,10
;-------------------GEOMETRY-----------------
gen 0,0 0,20 50,20 50,0 i=1,26 j=1,11
;-------------------BOUNDARY CONDITIONS--
---
fix x i=1 j=1,11
fix x i=26 j 1 11
fix x y j 1 i 1 26
;-------------------ELASTIC PROPERTIES------
prop bulk 19.2e6 shear 8.8e6 density 2000
notnull
;-------------------INITIAL CONDITION------
set g 9.81
solve
initial xdisp 0 ydisp 0
apply pressure 100e3 j 11 i 11 16
;-------------------CONST MODEL-----------------
model mohr i=1,25 j=1,10
;-------------------ELASTICOPLASTIC PROPERTIES
prop bulk 19.2e6 shear 8.8e6 density 2000
friction 20 cohesion 5e3 notnull
solve
Example 1: Shallow Footing
Example 1: Shallow Footing
Example 1.5: Circular Hole

;EXAMPLE 1.5
config
grid 20 20
model elastic
gen circle 10 10 2
gen adjust;

;--------------PROPERTIES--------------
prop shear=.3e8 bulk=1e8 density=1600
set g 9.81;

;--------------BOUNDARY CONDITIONS------
-
fix x i=1
fix x i=21
fix x y j=1
solve

;--------------EXCAVATION--------------
ini xdisp 0 ydisp 0
model null region i=10 j=11
model mohr notnull
prop shear=.3e8 bulk=1e8 density=1600
c=20e3 f=20 notnull
set large
solve
Example 1.5: Circular Hole

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4-5 Example 2: Stone Column Modeling

Stability study
• history unbal

Creating different layers
Example:
• group layer1 i=1,2 j=1,2
• prop b=1.6e7 s= 7.6e6 d=1800 group layer1

Creating stone column
Example:
• group stonecolumn i=1,2 j=1,2
• group cap i=1,2 j=1,2
• prop b=1.6e8 s= 7.6e7 d=2200 group stonecolumn
• prop b=1.6e8 s= 7.6e7 d=2200 group cap
• solve
• ini xdisp 0 ydisp=0
Different Settlement in the Model

![Graph showing different settlement in the model]
Example 2: Simplified Group Pile

;EXAMPLE 2 With Group Pile
config
grid 25 10
;#################################################################PHASE 1#################################################################
;---------------------CONST MODEL---------------------
model elastic i=1,25 j=1,10
;---------------------GEOMETRY---------------------
gen 0,0 0,20 50,20 50,0 i=1,26 j=1,11
;---------------------BOUNDARY CONDITIONS-----
fix x i=1 j=1,11
fix x i=26 j 1 11
fix x y j 1 i 1 26
;---------------------LAYERS---------------------
group layer1 j=1,7
group layer2 j=8,10
;---------------------ELASTIC PROPERTIES-----
prop bulk 19.2e6 shear 8.8e6 density 2000  group layer2
prop bulk 19.2e6 shear 8.8e6 density 2200  group layer1
;------------------INITIAL CONDITION-------
set g 9.81
solve
;#---------------------------------

;#---------------------------------
ini xdisp 0 ydisp 0
group pile i 7 j 8 10
group pile i 10 j 8 10
group pile i 13 j 8 10
group pile i 16 j 8 10
group pile i 19 j 8 10
prop bulk 20e7 shear 8e7 density 2500 group pile
solve
;#---------------------------------

;#---------------------------------
model mohr group layer1
model mohr group layer2

;------------------MOHR PROPERTIES-----
prop bulk 19.2e6 shear 8.8e6 density 2000 c 1e3 f 10 group layer2
prop bulk 19.2e6 shear 8.8e6 density 2200 c 30e3 f 30 group layer1

;------------------APPLYING PRESSUR-----
apply pressure 80000.0 from 7,11 to 20,11
solve
4-6 Example 3: Slope Stability
Example 3: Slope Stability

```plaintext
config
grid
gen
model elastic
table 1 (x y coordinates)
gen table 1
model null region 2 5
group soil notnull
prop dens bulk shear
fix
set gravity 9.81
solve
ini xdisp 0 ydisp 0
model mohr
prop dens bulk shear cohesion friction
solve
solve fos
set large
```
config
grid 30 19
model elastic
gen 0 0 0 240 400 240 400 0
table 1 0,100 100,120 150,190 190,200 210,230 290,235 310,240
gen table 1
table 2 0 37.89 280 180 350 240
gen table 2
model null region i=1 j=18
group sand region i 1 j 6
group clay region i 30 j 1
;
set g 32.3
fix x i 1
fix x i 31
fix x y j 1
prop b 5e5 s 2.3e5 d 4 group clay
prop b 1.67e5 s 7.69e5 d 3.7 group sand
solve
ini xdisp 0 ydisp 0
model mohr group clay
model mohr group sand
prop b 5e5 s 2.3e5 d 4 c 2000 f 10 group clay
prop b 1.67e5 s 7.69e5 d 3.7 c 500 f 25 group sand
his unbal
set large
step 100
;solve fos
4-7 Structural Element

- Beam
- Pile
- Cable
Nail and Anchor Modeling

Fig. 4) Mechanical representation of fully bonded reinforcement
Which accounts for shear behaviour of the grout annulus.

\[ Kg = \frac{2\pi G}{10\ln(1+2tD)} \quad cg = \pi c(D+2t) \]

Where \( G \) is shear modulus of the shear zone and is identical to shear modulus of soil;
\( t \) is annulus thickness of the shear zone and is considered equal to 0.004 m.

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Example 4: Soil Nailing System

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit (SI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nail diameter</td>
<td>20</td>
<td>mm</td>
</tr>
<tr>
<td>Drill hole diameter</td>
<td>100</td>
<td>mm</td>
</tr>
<tr>
<td>Young’s modulus of nail</td>
<td>200</td>
<td>GPa</td>
</tr>
<tr>
<td>Young’s modulus of grout</td>
<td>22</td>
<td>GPa</td>
</tr>
<tr>
<td>Young’s modulus of grouted nail</td>
<td>29.12</td>
<td>GPa</td>
</tr>
<tr>
<td>Annulus thickness</td>
<td>0.004</td>
<td>m</td>
</tr>
<tr>
<td>Shear zone cohesive strength</td>
<td>6.28</td>
<td>kPa</td>
</tr>
<tr>
<td>Shear zone friction angle</td>
<td>31</td>
<td>degree</td>
</tr>
<tr>
<td>Shear stiffness of the shear zone</td>
<td>219.6</td>
<td>MPa</td>
</tr>
<tr>
<td>Compressive yield strength of the grouted nail</td>
<td>30</td>
<td>MPa</td>
</tr>
<tr>
<td>Tensile yield strength of the grouted nail</td>
<td>30</td>
<td>MPa</td>
</tr>
<tr>
<td>Density of grouted nail</td>
<td>2200</td>
<td>kg/m³</td>
</tr>
</tbody>
</table>

Table 3. Structural parameters for shotcrete

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit (SI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>24.5</td>
<td>Gpa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>2200</td>
<td>kg/m³</td>
</tr>
</tbody>
</table>
Example 4: Soil Nailing System

struct node 1501 0.0 0.0
struct node 1500 0.0 -32.0
struct beam beg node 1500 end node 1501 prop 1001 seg 32
struct prop 1001
int 101 as from 31,35 to 31,67 bs from node 1500 to node 1501
int 102 as from 32,67 to 32,35 bs from node 1501 to node 1500
struct prop 1001 density 2400.0
struct cable begin node 1533 end node 1534 seg 1 tension 768000.0 prop 2001
struct cable begin node 1535 end node 1533 seg 6 prop 2002
struct prop 2002
struct prop 2001 spac 2.3 e 2.1E11 area 0.0015 yie 1e10 kb 0 sb 0
struct prop 2002 spac 2.3 e 2.1E11 area 0.0015 yie 1e10 kb 1.0E8 sb 1e8
4-8 Seismic Considerations

- Pseudo-Static Analysis
- Dynamic Analysis
Pseudo-Static vs. Dynamic Analysis

Seismic coefficients for pseudostatic slope analysis, Cristiano Melo and Sunil Sharma

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### Recommended Horizontal Seismic Coefficients

<table>
<thead>
<tr>
<th>Horizontal Seismic Coefficient, $k_h$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05 - 0.15</td>
<td>In the United States</td>
</tr>
<tr>
<td>0.12 - 0.25</td>
<td>In Japan</td>
</tr>
<tr>
<td>0.1</td>
<td>“severe” earthquakes</td>
</tr>
<tr>
<td>0.2</td>
<td>“violent, destructive” earthquakes</td>
</tr>
<tr>
<td></td>
<td>Terzaghi [4]</td>
</tr>
<tr>
<td>0.5</td>
<td>“catastrophic” earthquakes</td>
</tr>
<tr>
<td>0.1 - 0.2</td>
<td>Seed [2], $FOS \geq 1.15$</td>
</tr>
<tr>
<td>0.10</td>
<td>Major Earthquake, $FOS &gt; 1.0$</td>
</tr>
<tr>
<td></td>
<td>Corps of Engineers [5]</td>
</tr>
<tr>
<td>0.15</td>
<td>Great Earthquake, $FOS &gt; 1.0$</td>
</tr>
<tr>
<td>$\frac{1}{2}$ to $\frac{1}{3}$ of PHA</td>
<td>Marcuson [6], $FOS &gt; 1.0$</td>
</tr>
<tr>
<td>$\frac{1}{2}$ of PHA</td>
<td>Hynes-Griffin [7], $FOS &gt; 1.0$</td>
</tr>
</tbody>
</table>

Seismic coefficients for pseudostatic slope analysis, Cristiano Melo and Sunil Sharma

© Siavash Zamiran, Steven F. Bartlett, FLAC manual, Plaxis manual, 2016
Pseudo-Static Analysis

Example 5: Pseudostatic analysis of a slope (FOS)
Dynamic Analysis
Dynamic Analysis, Important points

- Loading
- Damping
- Boundary condition

In FLAC, the dynamic input can be applied in one of the following ways:

(a) an acceleration history;
(b) a velocity history;
(c) a stress (or pressure) history; or
(d) a force history.
Boundary Conditions

Figure 1.6  Model for seismic analysis of surface structures and free-field mesh
Boundary Conditions

(a) Flexible base

(a) Rigid base
Codes for Dynamic Analysis

config dyn
set dyn off
set dyn on
apply xquiet j=1
apply ff
apply nquiet squiet j=1 ;(bottom)
;------------------------damping
set dy_damp struc rayl 0.05 1.64 mass
set dy_damp rayl 0.05 1.64
apply sxy 4.8e5 hist table 1 j 1
set dytime 0
hist reset
hist dytime
his 10 xacc i=50 j=2
his 11 xacc i=50 j=26
solve dytime 4

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Chapter 5

Numerical Modeling in Plaxis
Main Window

- Main Menu
- Toolbar (General)
- Toolbar (Geometry)
- Ruler
- Draw area
- Origin
- Manual Input
- Cursor position indicator

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Property Information Box

**Mohr-Coulomb - Sand**

**Stiffness**
- \( E_{\text{ref}} \): 1.300E+04 kN/m²
- \( \nu (\text{nu}) \): 0.300

**Strength**
- \( c_{\text{ref}} \): 1.000 kN/m²
- \( \phi (\text{phi}) \): 31.000 °
- \( \psi (\text{psi}) \): 0.000 °

**Alternatives**
- \( G_{\text{ref}} \): 5000.000 kN/m²
- \( E_{\text{oed}} \): 1.750E+04 kN/m²

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General Setting Box

![General settings window]

- **Project**
  - Filename: `<NoName>`
  - Directory: `...`
  - Title: `<NoName>`

- **General**
  - Model: *Plane strain*
  - Elements: *15-Node*

- **Acceleration**
  - Gravity angle: `-90°`, `1.0 G`
  - x-acceleration: `0.000 G`
  - y-acceleration: `0.000 G`

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Genera Setting Tab Sheet

![Genera Setting Tab Sheet](image)

- **Units**
  - **Length**: m
  - **Force**: kN
  - **Time**: day

- **Geometry dimensions**
  - **Left**: 0.000 m
  - **Right**: 50.000 m
  - **Bottom**: 0.000 m
  - **Top**: 25.000 m

- **Grid**
  - **Spacing**: 1.000 m
  - **Number of intervals**: 1

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Example 5: Shallow Footing

Figure 3.1 Geometry of a circular footing on a sand layer

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Geometry Model
Axisymmetric Finite Element Mesh
Initial Principal Stress Tensors
Staged Construction

![Plaxis software interface showing staged construction parameters](image)

- **Control parameters**: Additional Steps = 250
- **Iterative procedure**: Standard setting
- **Loading input**: Time interval = 0.0000 day, Estimated end time = 0.0000 day
- **Initial phase**:
  - Phase no. = 0, Start from = 0, Calculation = N/A, Loading input = N/A
  - Time = 0.0, Water = 0, First = 0, Last = 0
- **<Phase 1>**:
  - Phase no. = 1, Start from = 0, Calculation = Plastic, Loading input = Staged construction
  - Time = 0.0, Water = 1, First = 1, Last = 73

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Deformed Mesh
Principle Stress After Loading
Thank You!